$\to \mathrm{Out}^{FC}(\Pi_1)$

$\overset{p_1}{\twoheadrightarrow} \Pi_1 \to 1$

Thus, we have

$$\begin{tikzcd} \mathbb{D} \arrow[r, phantom, "\subseteq"] \arrow[d] & \Pi_2 \arrow[r, "\alpha", "\sim"'] \arrow[d, "(p_1, p_2)"] & \Pi_2 \arrow[d, "(p_1, p_2)"] \\ \Pi_1 \arrow[r, phantom, "\subseteq"] & \Pi_1 \times \Pi_1 \arrow[r, "\sim", "(\alpha_1, \alpha_2)"'] & \Pi_1 \times \Pi_1 \end{tikzcd}$$

"diag"

$X_2 \hookrightarrow X \times X$ open imm

So it suffices to show that

<u>claim</u> $\alpha(\mathbb{D}) = \circledast^{-1} \mathbb{D} \circledast$

$\boxed{\mathrm{H}}_2 := \ker p_1 \cap \ker p_2 \subseteq \Pi_2$

Since
- $\alpha(\boxed{\mathrm{H}}_2) = \boxed{\mathrm{H}}_2 \supseteq \mathbb{I}$
- α : C-adm $\rightsquigarrow$ $\alpha(\mathbb{I})$: $\exists$ cusp inertia

it follows $\alpha(\mathbb{I}) = \circledast^{-1} \mathbb{I} \circledast$

By the fact "$N_{\Pi_2}(\mathbb{I}) = \mathbb{D}$", we have $\alpha(\mathbb{D}) = \circledast^{-1} \mathbb{D} \circledast$ //

Def X: arbitrary hyperbolic curve $/k$

$$\mathrm{Aut}^{\mathrm{IFC}}(\Pi_2) = \{ \alpha \in \mathrm{Aut}^{\mathrm{FC}}(\Pi_2) \mid \alpha_1 = \alpha_2 = \mathrm{id} \}$$

where
$$\begin{tikzcd} & \Pi_1 \arrow[loop right, "\alpha_1"] \\ \Pi_2 \arrow[loop left, "\alpha"] \arrow[ur, two heads, "p_1"] \arrow[dr, two heads, "p_2"'] & \\ & \Pi_1 \arrow[loop right, "\alpha_2"] \end{tikzcd}$$

Next month
2/16
13:00-
K. Nakamura

it suffic

Let

Th

In particular, we have a factorization

inj (∵ Π_2: center-free)

$$\begin{tikzcd} \boxed{\mathrm{H}}_{12} \arrow[r, hook] \arrow[d, equal] \arrow[drr, bend right] & \Pi_2 \arrow[r, hook, "\mathrm{Inn}"] \arrow[d, phantom, "\circlearrowright"] & \mathrm{Aut}(\Pi_2) \\ \mathrm{Ker}\, p_1 \cap \mathrm{Ker}\, p_2 & & \mathrm{Aut}^{\mathrm{IFC}}(\Pi_2) \arrow[u, hook] \end{tikzcd}$$

$\alpha_1 = \alpha_2 = id\}$

By t

Thm 1 (tripod case)

Suppose that $X := \mathbb{P}^1_k \setminus \{a, b, c\}$

Then (1) $\mathbb{I}_{12} \longrightarrow \mathrm{Aut}^{IFC}(\Pi_2)$ bijective

(2) $\mathrm{Out}^{FC}(\Pi_2) \longrightarrow \mathrm{Out}^{FC}(\Pi_1)$ injective

☺ First, we prove (1) ⇒ (2)

$\alpha \in \mathrm{Aut}^{FC}(\Pi_2)$ s.t. $\alpha_1 = \mathrm{Inn}(g_1)$ $\exists g_1 \in \Pi_1$

Note that, by lem 1, $\alpha_2 = \mathrm{Inn}(g_2)$ $\exists g_2 \in \Pi_1$

$\Pi_2 \xrightarrow{(p_1, p_2)} \Pi_1 \times \Pi_1$ $(\leftarrow X_2 \hookrightarrow X \times_k X)$

$\Downarrow$

$\exists g \longmapsto (g_1, g_2)$

Next month
2/16
13:00~
K.Nakamura

ecting

injective

$\exists g_1 \in \Pi_1$

$\exists g_2 \in \Pi_1$

$\therefore \mathrm{Inn}(g^{-1}) \circ \alpha \in \mathrm{Aut}^{\mathrm{IFC}}(\Pi_2) \Leftarrow \mathrm{H}_2$

$\therefore \alpha \in \mathrm{Inn}(\Pi_2)$

So, we prove (1)

Let $Z^{\log}$: smooth log curve $/k$ associated to X

$Z_n^{\log}$: n-th log config sp of $Z^{\log}$

[Note that $\Pi_n \cong \pi_1^{\log}(Z_n^{\log})$]

$$\Pi_{2/1} := \ker(\Pi_2 \xrightarrow{p_1} \Pi_1)$$

$$\alpha \in \mathrm{Aut}^{\mathrm{IFC}}(\Pi_2)$$

Now by considering the "geom gen fiber of $X_2 \to X$"

$$1 \to \Pi_{2/1} \to \Pi_2 \xrightarrow{p_1} \Pi_1 \to 1$$

$$\Pi_{2/1} \overset{p_2}{\twoheadrightarrow} \Pi_1$$

$$\Pi_{2/1} \cong \Pi_1\left(\text{—o—o—o—o— } \mathbb{P}^1\right)$$

$*$ (new cusp), a, b, c

Moreover, since • $\Pi_{2/1} \overset{p_2}{\twoheadrightarrow} \Pi_1$ induced by

open imm (—o—o—o—o— ↪ —o—o—o—)

I_*, I_a, I_b, I_c ; $*$, a, b, c ; a, b, c

• $\alpha_2 = id$ ($\alpha \in Aut^{IFC}$)

• α : C-admissible

I_a, I_b, I_c ; a, b, c ; $p_2 \downarrow$; I_a, I_b, I_c

It follows that

"α induces identity permutation on the set of conj class of cusp inertia subgps"

Next month 2/16 13:00- K.Nakamura

Note that

$$\Pi_2 \overset{(p_1, p_2)}{\twoheadrightarrow} \Pi_1 \ldots$$

$\exists\, g \longmapsto (\ldots$

In partic...

In particular,

If we fix a cusp inertia $I_a \subseteq \Pi_{2/1}$ associated to a

then $\alpha(I_a) = \xi^{-1} I_a \xi \qquad \xi \in \Pi_{2/1}$

$$p_2 \alpha(I_a) = p_2(\xi)^{-1} p_2(I_a)\, p_2(\xi)$$
$$\overset{\|}{p_2(I_a)}$$

$$\Rightarrow p_2(\xi) \in N_{\Pi_1}(p_2(I_a)) \underset{\uparrow}{=} p_2(I_a)$$

Prop 1, (2)

Thus, by replacing "ξ"

by an appropriate element,

we may assume that $\underline{\xi \in \Pi_{12}}$.

Next month
2/16
13:00~
K. Nakamura

ible

tion

f cusp inertia subgps"

F_b

Hence, by replacing

we may

Next, by consider

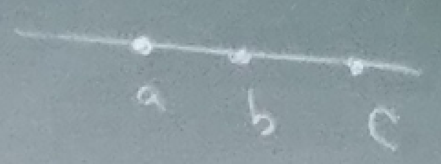
$\Pi_{2/1} \cong \Pi$

$\Pi_{F_b} \subseteq \Pi_{2/1}$

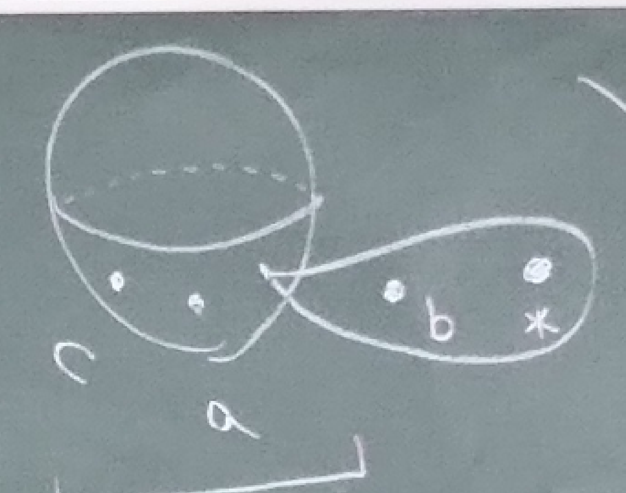
associated to a

Hence, by replacing "α" by "$\mathrm{Inn}(\xi') \circ \alpha$", $\xi^{\log}$

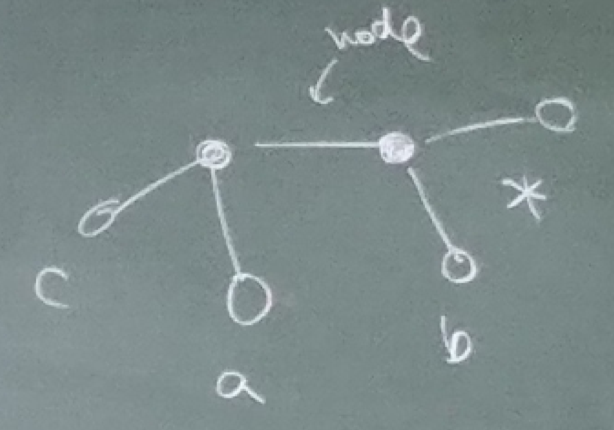
We may assume that $\alpha(I_a) = I_a$



Next, by considering the fiber of $\xi_2^{\log} \to \xi^{\log}$ over b

$\Pi_{2/1} \cong \Pi \Big($  $\Big)$

F_b



$\Pi_{F_b} \subseteq \Pi_{2/1}$: unique vertical subgp containing I_a

(among its $\Pi_{2/1}$-conj)

2)

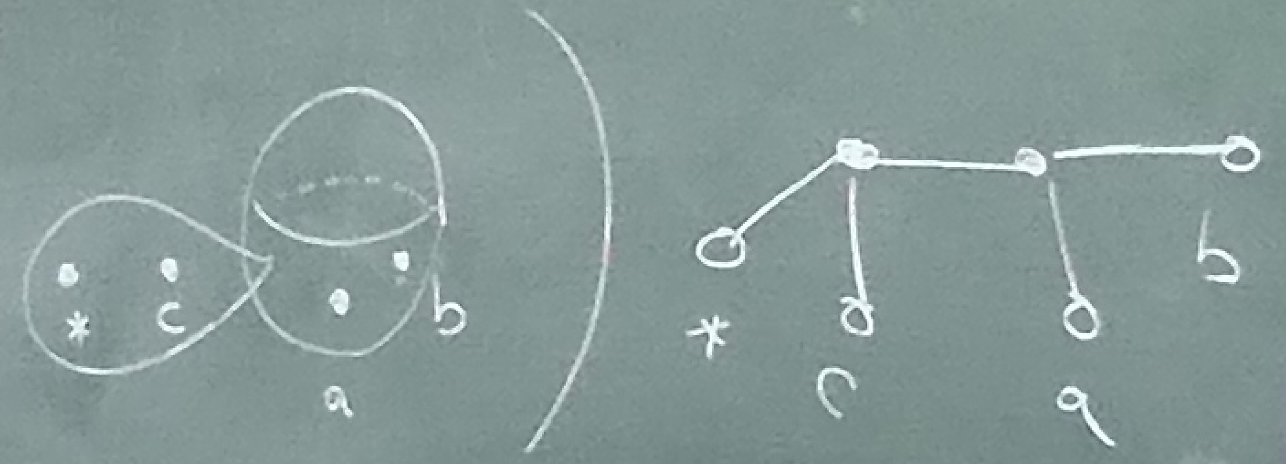
$\alpha(\Pi_{F_b}) \supseteq \alpha(I_a) = I_a$

By comb GC $\Rightarrow \alpha(\Pi_{F_b}) = \Pi_{F_b}$

Next month
2/16
13:00~
K. Nakamura

We may assume

Also, by considering the fiber of $\mathcal{Z}_2^{\log} \to \mathcal{Z}^{\log}$ over $\underset{\sim}{c}$

$\Pi_{2/1} \cong \Pi\Big($  $\Big)$

$\Rightarrow \alpha(\Pi_{F_c}) = \Pi_{F_c}$